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ABSTRACT

This volume provides the appendices to the report appearing as SE 020 236, which is a study of the effects of students' grade level (four, five, or six) and of five variables related to types of open arithmetical sentences on students' ability to solve open-sentence problems. (SD)

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Technical Report No. 375
(Part 2 of 2 Parts)

DIFFERENTIAL PERFORMANCE OF FOURTH- THROUGH SIXTH-GRADE STUDENTS IN SOLVING OPEN MULTIPLICATION AND DIVISION SENTENCES

by

Mary Jane McMaster

Report from the Project on
Conditions of School Learning and Instructional Strategies

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Center for Cognitive Learning
The University of Wisconsin
Madison, Wisconsin

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SE 020 237

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ABSTRACT

Differential Performance of Fourth- Through Sixth-Grade Students in Solving Open Multiplication and Division Sentences

Mary Jane McMaster

Under the Supervision of Professor J. Fred Weaver

The Problem

The purpose of this study was to find out whether differences exist in pupils' performance when solving selected types of open multiplication and division sentences derived from the form a o b = c.

Procedure

Specifically, this investigation sought to find out the differences in students' responses to open number sentences when the following factors were varied: (A) school grade (4, 5, and 6), (B) the symbol for the operation specified in a sentence (x or +), (C) sentence type as determined by the symmetric property of the equality relation (a o b = c versus c = a o b), (D) the position of the placeholder in a sentence (a, b, or c), (E) the existence or non-existence of an open sentence solution within the set of whole numbers (■ x b = 20 versus ■ x 5 = 21), and (F) the largest number being a basic fact product or not a basic fact product in open sentences which have no whole number solution (3 x ■ = 25 versus 3 x ■ = 23).

Two distinct kinds of multiplication and division open sentence tests were constructed and administered to 1298 fourth-, fifth-, and sixth-grade students from eight schools. Each student was administered a 28-item open sentence number puzzle test (NPT) and a 14-item basic multiplication and division test (BMDT).

The data furnished by all 1298 subjects were corrected and coded by the investigator. The information was then key punched for computer analysis and analyzed by the Fortap Statistical Package. This process yielded descriptive statistical results. The data furnished by students who responded correctly to at least four of the five nontrivial multiplication items and four of the five nontrivial items on the BMDT, were further analyzed by ANOVA and Wilcoxen Signed Ranks Test.

Results

1. The performance level of subjects on open sentences having whole number solutions was significantly different between grade levels.
2. The performance level of subjects on open multiplication sentences was significantly different from the performance level of subjects on open division sentences.
3. The performance level of subjects on operation-left open sentences was significantly different from the performance level of subjects on operation-right open sentences.

4. The performance level of subjects on open sentences was significantly different for placeholder positions a, b, and c.
5. Significant interactions existed among the following factors: grade level, operation, symmetric property, and placeholder position.
6. The performance level of subjects on open number sentences which have no whole number solutions was significantly different from the performance level of subjects on open sentences which have whole number solutions.
7. Relative to the open sentences with no whole number solutions, there was no significant difference between students' performance level on open sentences in which the largest number was a basic fact product, and students' performance level on open sentences in which the largest number was not a basic fact product.

Conclusion

The analysis was complex to interpret because of the significant interactions. There appeared to be a very high interaction between operation division and placeholder position a. Significant interactions also existed between the following factors: (1) grade and operation; (2) grade and symmetric factor; (3) grade and placeholder position; (4) operation and symmetric factor; (5) operation, symmetric factor, and grade; (6) operation and placeholder position; (7) symmetric factor and placeholder position; and (8) operation, symmetric factor, and placeholder position. Caution

must be exercised, therefore, in taking an overly simplistic interpretation of significant differences between levels of principal factors.

Nevertheless, the evidence warrants the belief that much greater attention needs to be given to principal factors B, C, D, and E in preparation of text materials and in instruction pertaining to open multiplication and division sentences.

Appendix A

NUMBER PUZZLE TESTS 1 - 4

Number Puzzle Test 1			Number Puzzle Test 2	
	Number fact	Generic form	Number fact	Generic form
1)	2 x 3 = 6	W- 1	2 x 4 = 8	W- 3
2)	2 x 7 = 14	W- 2	2 x 8 = 16	W- 4
3)	3 x 4 = 12	W- 3	3 x 5 = 15	N- 5 *
4)	3 x 8 = 24	W- 4	3 x 9 = 27	W- 6
5)	4 x 5 = 20	W- 5	4 x 6 = 24	W- 1
6)	4 x 9 = 36	W- 6	5 x 2 = 10	W- 2
7)	5 x 6 = 30	W- 1	5 x 7 = 35	W- 3
8)	6 x 2 = 12	W- 2	6 x 3 = 18	W- 4
9)	6 x 7 = 42	N- 3 #	6 x 8 = 48	W- 5
10)	7 x 3 = 21	N- 4 *	7 x 4 = 28	N- 6#
11)	7 x 8 = 56	W- 5	7 x 9 = 63	W- 1
12)	8 x 4 = 32	W- 6	8 x 5 = 40	W- 2
13)	8 x 9 = 72	W- 1	9 x 2 = 18	W- 3
14)	9 x 5 = 45	W- 2	9 x 6 = 54	W- 4
15)	10 ÷ 5 = 2	W- 7	12 ÷ 6 = 2	W- 9
16)	18 ÷ 9 = 2	W- 8	6 ÷ 2 = 3	N-10 #
17)	18 ÷ 6 = 3	W- 9	21 ÷ 7 = 3	W-11
18)	8 ÷ 2 = 4	W-10	12 ÷ 3 = 4	W-12
19)	28 ÷ 7 = 4	W-11	32 ÷ 8 = 4	W- 7
20)	15 ÷ 3 = 5	W-12	20 ÷ 4 = 5	W- 8
21)	40 ÷ 8 = 5	N- 7 #	45 ÷ 9 = 5	W- 9
22)	24 ÷ 4 = 6	N- 8 *	30 ÷ 5 = 6	W-10
23)	54 ÷ 9 = 6	W- 9	14 ÷ 2 = 7	W-11
24)	35 ÷ 5 = 7	W-10	42 ÷ 6 = 7	W-12
25)	16 ÷ 2 = 8	W-11	24 ÷ 3 = 8	W- 7
26)	48 ÷ 6 = 8	W-12	56 ÷ 7 = 8	W- 8
27)	27 ÷ 3 = 9	W- 7	36 ÷ 4 = 9	N- 9*
28)	63 ÷ 7 = 9	W- 8	72 ÷ 8 = 9	W-10

* product (or dividend) is a basic fact
# product (or dividend) is not a basic fact

Number Puzzle Test 3		Number Puzzle Test 4	
Number fact	Generic form	Number fact	Generic form
1) 2 x 5 = 10	W- 5	2 x 6 = 12	W- 1
2) 2 x 9 = 18	W- 6	3 x 2 = 6	W- 2
3) 3 x 6 = 18	W- 1	3 x 7 = 21	W- 3
4) 4 x 2 = 8	W- 2	4 x 3 = 12	W- 4
5) 4 x 7 = 28	N- 3 *	4 x 8 = 32	W- 5
6) 5 x 3 = 15	W- 4	5 x 4 = 20	N- 6 *
7) 5 x 8 = 40	W- 5	5 x 9 = 45	W- 1
8) 6 x 4 = 24	W- 6	6 x 5 = 30	W- 2
9) 6 x 9 = 54	W- 1	7 x 2 = 14	W- 3
10) 7 x 5 = 35	W- 2	7 x 6 = 42	W- 4
11) 8 x 2 = 16	W- 3	8 x 3 = 24	N- 5 #
12) 8 x 6 = 48	N- 4 #	8 x 7 = 56	W- 6
13) 9 x 3 = 27	W- 5	9 x 4 = 36	W- 1
14) 9 x 7 = 63	W- 6	9 x 8 = 72	W- 2
15) 6 ÷ 3 = 2	W-11	8 ÷ 4 = 2	W- 7
16) 14 ÷ 7 = 2	W-12	16 ÷ 8 = 2	W- 8
17) 12 ÷ 4 = 3	N- 7 #	15 ÷ 5 = 3	W- 9
18) 24 ÷ 8 = 3	W- 8	27 ÷ 9 = 3	N-10 *
19) 20 ÷ 5 = 4	W- 9	24 ÷ 6 = 4	W-11
20) 36 ÷ 9 = 4	W-10	10 ÷ 2 = 5	W-12
21) 30 ÷ 6 = 5	W-11	35 ÷ 7 = 5	W- 7
22) 12 ÷ 2 = 6	W-12	18 ÷ 3 = 6	W- 8
23) 42 ÷ 7 = 6	W- 7	48 ÷ 8 = 6	N- 9 #
24) 21 ÷ 3 = 7	N- 8 *	28 ÷ 4 = 7	W-10
25) 56 ÷ 8 = 7	W- 9	63 ÷ 9 = 7	W-11
26) 32 ÷ 4 = 8	W-10	40 ÷ 5 = 8	W-12
27) 72 ÷ 9 = 8	W-11	18 ÷ 2 = 9	W- 7
28) 45 ÷ 5 = 9	W-12	54 ÷ 6 = 9	W- 8

* product (or dividend) is a basic fact
\# product (or dividend) is not a basic fact

Appendix B

NUMBER PUZZLE TESTS 1 - 4

Number Puzzle Test 1

Number fact	Generic form	Open sentence formed
1) 40 ÷ 8 = 5	N- 7	41 ÷ 8 = □
2) 18 ÷ 6 = 3	W- 9	18 ÷ □ = 3
3) 4 x 9 = 36	W- 6	36 = □ x 9
4) 8 ÷ 2 = 4	W-10	4 = 8 ÷ □
5) 28 ÷ 7 = 4	W-11	□ ÷ 7 = 4
6) 9 x 5 = 45	W- 2	□ = 9 x 5
7) 6 x 2 = 12	W- 2	□ = 6 x 2
8) 3 x 8 = 24	W- 4	24 = 3 x □
9) 5 x 6 = 30	W- 1	5 x 6 = □
10) 48 ÷ 6 = 8	W-12	8 = □ ÷ 6
11) 6 x 7 = 42	N- 3	6 x □ = 43
12) 10 ÷ 5 = 2	W- 7	10 ÷ 5 = □
13) 8 x 9 = 72	W- 1	8 x 9 = □
14) 16 ÷ 2 = 8	W-11	□ ÷ 2 = 8
15) 54 ÷ 9 = 6	W- 9	54 ÷ □ = 6
16) 63 ÷ 7 = 9	W- 8	□ = 63 ÷ 7
17) 35 ÷ 5 = 7	W-10	7 = 35 ÷ □
18) 7 x 3 = 21	N- 4	22 = 7 x □
19) 7 x 8 = 56	W- 5	□ x 8 = 56
20) 8 x 4 = 32	W- 6	32 = □ x 4
21) 2 x 3 = 6	W- 1	2 x 3 = □
22) 4 x 5 = 20	W- 5	□ x 5 = 20
23) 27 ÷ 3 = 9	W- 7	27 ÷ 3 = □
24) 3 x 4 = 12	W- 3	3 x □ = 12
25) 15 ÷ 3 = 5	W-12	5 = □ ÷ 3
26) 18 ÷ 9 = 2	W- 8	□ = 18 ÷ 9
27) 24 ÷ 4 = 6	N- 8	□ = 25 ÷ 4
28) 2 x 7 = 14	W- 2	□ = 2 x 7

Number Puzzle Test 2

Number fact	Generic form	Open sentence formed
1) 14 ÷ 2 = 7	W-11	□ ÷ 2 = 7
2) 7 x 9 = 63	W- 1	7 x 9 = □
3) 24 ÷ 3 = 8	W- 7	24 ÷ 3 = □
4) 5 x 2 = 10	W- 2	□ = 5 x 2
5) 72 ÷ 8 = 9	W-10	9 = 72 ÷ □
6) 20 ÷ 4 = 5	W- 8	□ = 20 ÷ 4
7) 9 x 2 = 18	W- 3	9 x □ = 18
8) 8 x 5 = 40	W- 2	□ = 8 x 5
9) 7 x 4 = 28	N- 6 #	29 = □ x 4
10) 3 x 5 = 15	N- 5 *	□ x 5 = 16
11) 5 x 7 = 35	W- 3	5 x □ = 35
12) 6 ÷ 2 = 3	N-10 #	3 = 7 ÷ □
13) 32 ÷ 8 = 4	W- 7	32 ÷ 8 = □
14) 2 x 8 = 16	W- 4	16 = 2 x □
15) 45 ÷ 9 = 5	W- 9	45 ÷ □ = 5
16) 30 ÷ 5 = 6	W-10	6 = 30 ÷ □
17) 3 x 9 = 27	W- 6	27 = □ x 9
18) 12 ÷ 6 = 2	W- 9	12 ÷ □ = 2
19) 12 ÷ 3 = 4	W-12	4 = □ ÷ 3
20) 6 x 8 = 48	W- 5	□ x 8 = 48
21) 6 x 3 = 18	W- 4	18 = 6 x □
22) 21 ÷ 7 = 3	W-11	□ ÷ 7 = 3
23) 2 x 4 = 8	W- 3	2 x □ = 8
24) 56 ÷ 7 = 8	W- 8	□ = 56 ÷ 7
25) 4 x 6 = 24	W- 1	4 x 6 = □
26) 9 x 6 = 54	W- 4	54 = 9 x □
27) 42 ÷ 6 = 7	W-12	7 = □ ÷ 6
28) 36 ÷ 4 = 9	N- 9 *	35 ÷ □ = 9

* indicates basic fact
\# indicates not a basic fact

Number Puzzle Test 3

	Number fact	Generic form	Open sentence formed
1)	8 x 2 = 16	W- 3	8 x □ = 16
2)	45 ÷ 5 = 9	W-12	9 = □ ÷ 5
3)	2 x 9 = 18	W- 6	18 = □ x 9
4)	21 ÷ 3 = 7	N- 8 *	□ = 20 ÷ 3
5)	56 ÷ 8 = 7	W- 9	56 ÷ □ = 7
6)	8 x 6 = 48	N- 4 #	47 = 8 x □
7)	72 ÷ 9 = 8	W-11	□ ÷ 9 = 8
8)	3 x 6 = 18	W- 1	3 x 6 = □
9)	6 x 9 = 54	W- 1	6 x 9 = □
10)	5 x 8 = 40	W- 5	□ x 8 = 40
11)	2 x 5 = 10	W- 5	□ x 5 = 10
12)	30 ÷ 6 = 5	W-11	□ ÷ 6 = 5
13)	42 ÷ 7 = 6	W- 7	42 ÷ 7 = □
14)	6 x 4 = 24	W- 6	24 = □ x 4
15)	4 x 7 = 28	N- 3 *	4 x □ = 27
16)	20 ÷ 5 = 4	W- 9	20 ÷ □ = 4
17)	4 x 2 = 8	W- 2	□ = 4 x 2
18)	14 ÷ 7 = 2	W-12	2 = □ ÷ 7
19)	24 ÷ 8 = 3	W- 8	□ = 24 ÷ 8
20)	36 ÷ 9 = 4	W-10	4 = 36 ÷ □
21)	32 ÷ 4 = 8	W-10	8 = 32 ÷ □
22)	9 x 3 = 27	W- 5	□ x 3 = 27
23)	7 x 5 = 35	W- 2	□ = 7 x 5
24)	12 ÷ 2 = 6	W-12	6 = □ ÷ 2
25)	9 x 7 = 63	W- 6	63 = □ x 7
26)	5 x 3 = 15	W- 4	15 = 5 x □
27)	12 ÷ 4 = 3	N- 7 #	13 ÷ 4 = □
28)	6 ÷ 3 = 2	W-11	□ ÷ 3 = 2

* indicates basic fact
\# indicates not a basic fact

Number Puzzle Test 4

Number fact	Generic form	Open sentence formed
1) 18 ÷ 2 = 9	W- 7	18 ÷ 2 = □
2) 3 x 2 = 6	W- 2	□ = 3 x 2
3) 48 ÷ 8 = 6	N- 9 #	47 ÷ □ = 6
4) 15 ÷ 5 = 3	W- 9	15 ÷ □ = 3
5) 18 ÷ 3 = 6	W- 8	□ = 18 ÷ 3
6) 8 x 3 = 24	N- 5 #	□ x 3 = 23
7) 3 x 7 = 21	W- 3	3 x □ = 21
8) 5 x 9 = 45	W- 1	5 x 9 = □
9) 6 x 5 = 30	W- 2	□ = 6 x 5
10) 54 ÷ 6 = 9	W- 8	□ = 54 ÷ 6
11) 9 x 8 = 72	W- 2	□ = 9 x 8
12) 4 x 8 = 32	W- 5	□ x 8 = 32
13) 7 x 6 = 42	W- 4	42 = 7 x □
14) 16 ÷ 8 = 2	W- 8	□ = 16 ÷ 8
15) 8 ÷ 4 = 2	W- 7	8 ÷ 4 = □
16) 7 x 2 = 14	W- 3	7 x □ = 14
17) 2 x 6 = 12	W- 1	2 x 6 = □
18) 40 ÷ 5 = 8	W-12	8 = □ ÷ 5
19) 28 ÷ 4 = 7	W-10	7 = 28 ÷ □
20) 9 x 4 = 36	W- 1	9 x 4 = □
21) 4 x 3 = 12	W- 4	12 = 4 x □
22) 63 ÷ 9 = 7	W-11	□ ÷ 9 = 7
23) 24 ÷ 6 = 4	W-11	□ ÷ 6 = 4
24) 10 ÷ 2 = 5	W-12	5 = □ ÷ 2
25) 8 x 7 = 56	W- 6	56 = □ x 7
26) 27 ÷ 9 = 3	N-10 *	3 = 28 ÷ □
27) 5 x 4 = 20	N- 6 *	21 = □ x 4
28) 35 ÷ 7 = 5	W- 7	35 ÷ 7 = □

* indicates basic fact
\# indicates not a basic fact

Appendix C

BASIC MULTIPLICATION AND DIVISION TEST COMBINATIONS

	TEST 1	TEST 2	TEST 3	TEST 4
1)	4 x 9	6 x 3	7 x 5	4 x 3
2)	8 x 1	7 ÷ 1	4 ÷ 1	3 x 1
3)	16 ÷ 2	72 ÷ 8	4 x 7	54 ÷ 6
4)	2 x 3	21 ÷ 7	2 x 9	63 ÷ 9
5)	8 x 9	3 x 1	32 ÷ 4	6 x 5
6)	3 x 8	4 x 6	1 x 3	4 ÷ 1
7)	8 ÷ 2	56 ÷ 7	30 ÷ 6	48 ÷ 8
8)	1 x 3	9 x 6	12 ÷ 2	7 ÷ 1
9)	15 ÷ 3	4 ÷ 1	20 ÷ 5	1 x 8
10)	4 ÷ 1	5 x 7	8 x 1	7 x 2
11)	3 x 4	30 ÷ 5	9 x 7	8 x 3
12)	18 ÷ 9	8 x 5	72 ÷ 9	5 x 4
13)	7 ÷ 1	32 ÷ 8	7 ÷ 1	16 ÷ 8
14)	54 ÷ 9	1 x 8	8 x 6	15 ÷ 5

Appendix D

PILOT STUDY RESULTS

Factor and Level					
Factor E	Factor C	Factor B	Factor D	Number of Items	Number of Incorrect Responses
A Solution Exists Within W	Mult.	a o b = c	a	2	5
			b	1	0
			c	3	2
		c = a o b	a	2	4
			b	1	4
			c	3	1
	Division	a o b = c	a	2	32
			b	2	7
			c	2	8
		c = a o b	a	2	36
			b	2	7
			c	2	7
No Solution Exists Within W	Mult.	a o b = c	b(NBF)	1	4
		c = a o b	b(BF)	1	1
	Division	a o b = c	c(NBF)	1	3
		c = a o b	c(BF)	1	3

Sample Size = 22 Students (fifth-grade)

W = Whole Numbers

NBF = Not a Basic Fact / BF = Basic Fact

March 1974

Appendix E

TEACHER QUESTIONNAIRE

MATH TEACHER _____

SCHOOL _____ GRADE _____

Please circle the appropriate letter to indicate the experience your students have had with each type of equation. Since all the basic facts will be used in this study, please focus on the format of the equation and not on the size of the numerals. (L indicates little experience, M indicates moderate experience, E indicates extensive experience)

a. $7 \times 8 = \square$ L M E

b. $\square = 5 \times 6$ L M E

c. $6 \times \square = 42$ L M E

d. $35 = 5 \times \square$ L M E

e. $\square \times 9 = 63$ L M E

f. $40 = \square \times 8$ L M E

g. $\square \div 9 = 8$ L M E

h. $6 = \square \div 8$ L M E

i. $30 \div \square = 5$ L M E

j. $6 = 42 \div \square$ L M E

k. $56 \div 7 = \square$ L M E

l. $\square = 35 \div 7$ L M E

m. equations which have no whole number solutions - for example,

$\square \times 9 = 29$ L M E

$3 = 28 \div \square$ L M E

Appendix F

FREQUENCY OF CODED RESPONSES 1-7 FOR WHOLE NUMBER SOLUTION CELLS

Explanation of responses 1-7

1. Correct solution
2. Subject responded with N, indicating no whole number solution.
3. Subject used the inverse operation, i.e., multiplied while solving a division puzzle or vice versa.
4. Subject responded with a numeral quite close to the correct solution.
5. Subject 'performed the operation across the equality sign.' For example, the puzzle 8 x $\square$ = 16 received 128 as the correct solution, indicating the subject might have changed the puzzle to the form 8 x 16 = $\square$.
6. Subject responded with some numeral not falling into any of the above categories.
7. The subject performed 'the inverse operation across the equality sign'. For example the puzzle 15 ÷ $\square$ = 3 received 45 for the solution, indicating the subject might have changed the puzzle to the form 3 x 15 = $\square$.

Note: - A zero coding was given to those puzzles to which the student did not respond or to which the response was illegible. It was necessary to employ the zero coding very rarely. If one sums the seven percentage scores on any puzzle, the total will be close to 99%. The difference between 99% and 100% is caused by the zero category being eliminated. In cases where no zero codes were employed, the sum of the seven percentages should be 100%.

Frequency of Coded Responses 1 - 7 For Whole Number Solution Cells

		Response	Number Giving this Response	Percent
Cell 1 MLA	☐ x 8 = 56	1	286	87.20
		2	15	4.57
		3	0	0.00
		4	11	3.35
		5	0	0.00
		6	9	2.74
		7	0	0.00
	☐ x 5 = 20	1	310	94.51
		2	3	0.91
		3	0	0.00
		4	3	0.91
		5	1	0.30
		6	6	1.83
		7	0	0.00
	☐ x 8 = 48	1	287	85.16
		2	16	4.75
		3	0	0.00
		4	7	2.08
		5	0	0.00
		6	21	6.23
		7	0	0.00
	☐ x 8 = 40	1	298	93.12
		2	10	3.13
		3	0	0.00
		4	3	0.94
		5	0	0.00
		6	8	2.50
		7	0	0.00
	☐ x 5 = 10	1	307	95.94
		2	1	0.31
		3	0	0.00
		4	0	0.00
		5	4	1.25
		6	7	2.19
		7	0	0.00
	☐ x 3 = 27	1	275	85.94
		2	19	5.94
		3	0	0.00
		4	6	1.87
		5	0	0.00
		6	16	5.00
		7	0	0.00

		Response	Number Giving This Response	Percent
Cell 1 (Cont.)	□ x 8 = 32	1	288	92.01
		2	10	3.19
		3	0	0.00
		4	5	1.60
		5	1	0.32
		6	8	2.56
		7	0	0.00
Cell 2 MLB	3 x □ = 12	1	306	93.29
		2	5	1.52
		3	0	0.00
		4	1	0.30
		5	1	0.30
		6	10	3.05
		7	0	0.00
	3 x □ = 21	1	293	93.61
		2	6	1.92
		3	0	0.00
		4	2	0.64
		5	0	0.00
		6	10	3.19
		7	0	0.00
	7 x □ = 14	1	298	95.21
		2	0	0.00
		3	0	0.00
		4	0	0.00
		5	1	0.32
		6	11	3.51
		7	0	0.00
	9 x □ = 18	1	318	94.36
		2	2	0.59
		3	0	0.00
		4	1	0.30
		5	0	0.00
		6	15	4.45
		7	0	0.00
	5 x □ = 35	1	312	92.58
		2	5	1.48
		3	0	0.00
		4	12	3.56
		5	0	0.00
		6	7	2.08
		7	0	0.00

		Response	Number Giving This Response	Percent
Cell 2 (Cont.)	2 x □ = 8	1	315	93.47
		2	2	0.59
		3	0	0.00
		4	2	0.59
		5	7	2.08
		6	8	2.37
		7	0	0.00
	8 x □ = 16	1	307	95.94
		2	3	0.94
		3	0	0.00
		4	1	0.31
		5	0	0.00
		6	9	2.81
		7	0	0.00
Cell 3 MLC	5 x 6 = □	1	319	97.26
		2	3	0.91
		3	0	0.00
		4	1	0.30
		5	0	0.00
		6	3	0.91
		7	0	0.00
	8 x 9 = □	1	296	90.24
		2	8	2.44
		3	0	0.00
		4	10	3.05
		5	0	0.00
		6	11	3.35
		7	0	0.00
	2 x 3 = □	1	314	95.73
		2	4	1.22
		3	0	0.00
		4	2	0.61
		5	0	0.00
		6	5	1.52
		7	0	0.00
	7 x 9 = □	1	291	86.35
		2	4	1.19
		3	0	0.00
		4	24	7.12
		5	0	0.00
		6	16	4.75
		7	0	0.00

		Response	Number Giving This Response	Percent
Cell 3 (Cont.)	4 x 6 = ☐	1	303	89.91
		2	8	2.37
		3	0	0.00
		4	9	2.67
		5	0	0.00
		6	14	4.15
		7	0	0.00
	3 x 6 = ☐	1	300	93.75
		2	0	0.00
		3	3	0.94
		4	5	1.56
		5	0	0.00
		6	12	3.75
		7	0	0.00
	6 x 9 = ☐	1	283	88.44
		2	3	0.94
		3	0	0.00
		4	24	7.50
		5	0	0.00
		6	9	2.81
		7	0	0.00
	5 x 9 = ☐	1	298	95.21
		2	1	0.32
		3	0	0.00
		4	9	2.88
		5	0	0.00
		6	4	1.28
		7	0	0.00
	2 x 6 = ☐	1	287	91.69
		2	1	0.32
		3	20	6.39
		4	0	0.00
		5	0	0.00
		6	3	0.96
		7	0	0.00
	9 x 4 = ☐	1	292	93.29
		2	1	0.32
		3	0	0.00
		4	7	2.24
		5	0	0.00
		6	12	3.83
		7	0	0.00

Cell 4
MRA

	Response	Number Giving This Response	Percent
36 = □ x 9	1	286	87.20
	2	13	3.96
	3	0	0.00
	4	4	1.22
	5	0	0.00
	6	22	6.71
	7	0	0.00
32 = □ x 4	1	286	87.20
	2	15	4.57
	3	0	0.00
	4	10	3.05
	5	0	0.00
	6	12	3.66
	7	0	0.00
27 = □ x 9	1	305	90.50
	2	14	4.15
	3	0	0.00
	4	6	1.78
	5	0	0.00
	6	11	3.26
	7	0	0.00
18 = □ x 9	1	303	94.69
	2	1	0.31
	3	0	0.00
	4	0	0.00
	5	0	0.00
	6	16	5.00
	7	0	0.00
24 = □ x 4	1	290	90.63
	2	9	2.81
	3	0	0.00
	4	4	1.25
	5	0	0.00
	6	16	5.00
	7	0	0.00
63 = □ x 7	1	275	85.94
	2	21	6.56
	3	0	0.00
	4	8	2.50
	5	0	0.00
	6	13	4.06
	7	0	0.00

		Response	Number Giving This Response	Percent
Cell 4 (Cont.)	56 = □ x 7	1	264	84.35
		2	14	4.47
		3	0	0.00
		4	19	6.07
		5	0	0.00
		6	15	4.79
		7	0	0.00
Cell 5 MRB	24 = 3 x □	1	280	85.37
		2	21	6.40
		3	0	0.00
		4	3	0.91
		5	1	0.30
		6	21	6.40
		7	0	0.00
	16 = 2 x □	1	315	93.47
		2	2	0.59
		3	0	0.00
		4	2	0.59
		5	2	0.59
		6	14	4.15
		7	0	0.00
	18 = 6 x □	1	271	80.42
		2	24	7.12
		3	0	0.00
		4	32	9.50
		5	0	0.00
		6	8	2.37
		7	0	0.00
	54 = 9 x □	1	289	85.76
		2	16	4.75
		3	0	0.00
		4	18	5.34
		5	0	0.00
		6	12	3.56
		7	0	0.00
	15 = 5 x □	1	309	96.56
		2	1	0.31
		3	0	0.00
		4	1	0.31
		5	1	0.31
		6	7	2.19
		7	0	0.00

		Response	Number Giving This Response	Percent
Cell 5 (Cont.)	42 = 7 x ☐	1	281	89.78
		2	10	3.19
		3	0	0.00
		4	5	1.60
		5	1	0.32
		6	14	4.47
		7	0	0.00
	12 = 4 x ☐	1	281	89.78
		2	6	1.92
		3	0	0.00
		4	5	1.60
		5	9	2.88
		6	10	3.19
		7	0	0.00
Cell 6 MRC	☐ = 9 x 5	1	302	92.07
		2	10	3.05
		3	0	0.00
		4	6	1.83
		5	0	0.00
		6	8	2.44
		7	0	0.00
	☐ - 6 x 2	1	303	92.38
		2	3	0.91
		3	15	4.57
		4	1	0.30
		5	0	0.00
		6	5	1.52
		7	0	0.00
	☐ = 2 x 7	1	307	93.60
		2	11	3.35
		3	0	0.00
		4	0	0.00
		5	0	0.00
		6	7	2.13
		7	0	0.00
	☐ = 5 x 2	1	325	96.44
		2	6	1.78
		3	0	0.00
		4	0	0.00
		5	0	0.00
		6	5	1.48
		7	0	0.00

		Response	Number Giving This Response	Percent
Cell 6 (Cont.)	□ = 8 x 5	1	313	92.88
		2	6	1.78
		3	0	0.00
		4	12	3.56
		5	0	0.00
		6	5	1.48
		7	0	0.00
	□ = 4 x 2	1	302	94.37
		2	2	0.62
		3	10	3.13
		4	2	0.62
		5	0	0.00
		6	3	0.94
		7	0	0.00
	□ = 7 x 5	1	299	93.44
		2	8	2.50
		3	0	0.00
		4	7	2.19
		5	0	0.00
		6	5	1.56
		7	0	0.00
	□ = 3 x 2	1	300	95.85
		2	5	1.60
		3	0	0.00
		4	3	0.96
		5	0	0.00
		6	5	1.60
		7	0	0.00
	□ = 6 x 5	1	303	96.81
		2	3	0.96
		3	0	0.00
		4	1	0.32
		5	0	0.00
		6	5	1.60
		7	0	0.00
	□ = 9 x 8	1	294	93.93
		2	3	0.96
		3	0	0.00
		4	5	1.60
		5	0	0.00
		6	9	2.88
		7	0	0.00

		Response	Number Giving This Response	Percent
Cell 7 DLA	□ ÷ 7 = 4	1	122	37.20
		2	167	50.91
		3	0	0.00
		4	0	0.00
		5	0	0.00
		6	36	10.98
		7	0	0.00
	□ ÷ 2 = 8	1	162	49.39
		2	23	7.01
		3	0	0.00
		4	0	0.00
		5	128	39.02
		6	11	3.35
		7	0	0.00
	□ ÷ 2 = 7	1	150	44.51
		2	154	45.70
		3	0	0.00
		4	0	0.00
		5	0	0.00
		6	28	8.31
		7	0	0.00
	□ ÷ 7 = 3	1	173	51.34
		2	132	39.17
		3	0	0.00
		4	1	0.30
		5	0	0,00
		6	26	7.72
		7	0	0.00
	□ ÷ 9 = 8	1	121	37.81
		2	173	54.06
		3	0	0.00
		4	4	1.25
		5	0	0.00
		6	18	5.62
		7	0	0.00
	□ ÷ 6 = 5	1	132	41.25
		2	170	53.13
		3	0	0.00
		4	0	0.00
		5	0	0.00
		6	16	5.00
		7	0	0.00

		Response	Number Giving This Response	Percent
Cell 7 (Cont.)	□ ÷ 3 = 2	1	179	55.94
		2	124	38.75
		3	0	0.00
		4	3	0.94
		5	0	0.00
		6	12	3.75
		7	0	0.00
	□ ÷ 9 = 7	1	137	43.77
		2	138	44.09
		3	0	0.00
		4	4	1.28
		5	0	0.00
		6	31	9.90
		7	0	0.00
	□ ÷ 6 = 4	1	161	51.44
		2	119	38.02
		3	0	0.00
		4	2	0.64
		5	0	0.00
		6	28	8.95
		7	0	0.00
Cell 8 DLB	18 ÷ □ = 3	1	297	90.55
		2	16	4.88
		3	0	0.00
		4	0	0.00
		5	0	0.00
		6	12	3.66
		7	0	0.00
	54 ÷ □ = 6	1	265	80.79
		2	29	8.84
		3	0	0.00
		4	8	2.44
		5	0	0.00
		6	20	6.10
		7	1	0.30
	45 ÷ □ = 5	1	304	90.21
		2	13	3.86
		3	0	0.00
		4	10	2.97
		5	0	0.00
		6	9	2.67
		7	0	0.00

		Response	Number Giving This Response	Percent
Cell 8 (Cont.)	12 ÷ □ = 2	1	309	91.69
		2	11	3.26
		3	0	0.00
		4	1	0.30
		5	0	0.00
		6	12	3.56
		7	2	0.59
	56 ÷ □ = 7	1	273	85.31
		2	16	5.00
		3	0	0.00
		4	18	5.62
		5	0	0.00
		6	12	3.75
		7	0	0.00
	20 ÷ □ = 4	1	287	89.69
		2	14	4.37
		3	0	0.00
		4	12	3.75
		5	0	0.00
		6	4	1.25
		7	2	0.62
	15 ÷ □ = 3	1	298	95.21
		2	4	1.28
		3	0	0.00
		4	0	0.00
		5	0	0.00
		6	7	2.24
		7	3	0.96
Cell 9 DLC	10 ÷ 5 = □	1	277	84.45
		2	3	0.91
		3	15	4.57
		4	0	0.00
		5	0	0.00
		6	30	9.15
		7	0	0.00
	27 ÷ 3 = □	1	262	79.88
		2	29	8.84
		3	0	0.00
		4	4	1.22
		5	0	0.00
		6	26	7.93
		7	0	0.00

		Response	Number Giving This Response	Percent
Cell 9 (Cont.)	24 ÷ 3 = ☐	1	297	88.13
		2	11	3.26
		3	2	0.59
		4	10	2.97
		5	0	0.00
		6	16	4.75
		7	0	0.00
	32 ÷ 8 = ☐	1	308	91.39
		2	7	2.08
		3	1	0.30
		4	5	1.48
		5	0	0.00
		6	14	4.15
		7	0	0.00
	42 ÷ 7 = ☐	1	287	89.69
		2	7	2.19
		3	0	0.00
		4	8	2.50
		5	0	0.00
		6	16	5.00
		7	0	0.00
	18 ÷ 2 = ☐	1	267	85.30
		2	1	0.32
		3	5	1.60
		4	9	2.88
		5	0	0.00
		6	31	9.90
		7	0	0.00
	8 ÷ 4 = ☐	1	270	86.26
		2	2	0.64
		3	20	6.39
		4	0	0.00
		5	0	0.00
		6	17	5.43
		7	0	0.00
	35 ÷ 7 = ☐	1	286	91.37
		2	8	2.56
		3	1	0.32
		4	2	0.64
		5	0	0.00
		6	12	3.83
		7	0	0.00

		Response	Number Giving This Response	Percent
Cell 10 DRA	8 = □ ÷ 6	1	87	26.52
		2	199	60.67
		3	0	0.00
		4	2	0.61
		5	0	0.00
		6	36	10.98
		7	0	0.00
	5 = □ ÷ 3	1	102	31.10
		2	180	54.88
		3	0	0.00
		4	0	0.00
		5	0	0.00
		6	40	12.20
		7	0	0.00
	4 = □ ÷ 3	1	117	34.72
		2	185	54.90
		3	0	0.00
		4	0	0.00
		5	0	0.00
		6	29	8.61
		7	0	0.00
	7 = □ ÷ 6	1	102	30.27
		2	200	59.35
		3	0	0.00
		4	1	0.30
		5	0	0.00
		6	29	8.61
		7	0	0 00
	9 = □ ÷ 5	1	89	27.81
		2	197	61.56
		3	0	0.00
		4	2	0.62
		5	0	0.00
		6	29	9.06
		7	0	0.00
	2 = □ ÷ 7	1	108	33.75
		2	189	59.06
		3	0	0.00
		4	0	0.00
		5	0	0.00
		6	19	5.94
		7	0	0.00

		Response	Number Giving This Response	Percent
Cell 10 (Cont.)	6 = □ ÷ 2	1	120	37.50
		2	28	8.75
		3	0	0.00
		4	0	0.00
		5	160	50.00
		6	9	2.81
		7	0	0.00
	8 = □ ÷ 5	1	83	26.52
		2	199	63.58
		3	0	0.00
		4	1	0.32
		5	0	0.00
		6	26	8.31
		7	0	0.00
	5 = □ ÷ 2	1	116	37.06
		2	162	51.76
		3	0	0.00
		4	0	0.00
		5	0	0.00
		6	32	10.22
		7	0	0.00
Cell 11 DRB	4 = 8 ÷ □	1	227	69.21
		2	45	13.72
		3	0	0.00
		4	0	0.00
		5	0	0.00
		6	23	7.01
		7	30	9.15
	7 = 35 ÷ □	1	282	85.98
		2	17	5.18
		3	0	0.00
		4	4	1.22
		5	0	0.00
		6	14	4.27
		7	5	1.52
	9 = 72 ÷ □	1	297	88.13
		2	17	5.04
		3	0	0.00
		4	13	3.86
		5	0	0.00
		6	7	2.08
		7	0	0.00

		Response	Number Giving This Response	Percent
Cell 11 (Cont.)	6 = 30 ÷ □	1	296	87.83
		2	21	6.23
		3	0	0.00
		4	4	1.19
		5	0	0.00
		6	11	3.26
		7	1	0.30
	4 = 36 ÷ □	1	265	82.81
		2	26	8.12
		3	0	0.00
		4	11	3.44
		5	0	0.00
		6	15	4.69
		7	1	0.31
	8 = 32 ÷ □	1	277	86.56
		2	18	5.62
		3	0	0.00
		4	4	1.25
		5	0	0.00
		6	20	6.25
		7	0	0.00
	7 = 28 ÷ □	1	273	87.22
		2	22	7.03
		3	0	0.00
		4	6	1.92
		5	0	0.00
		6	8	2.56
		7	1	0.32
Cell 12 DRC	□ = 24 ÷ 8	1	285	89.06
		2	17	5.31
		3	0	0.00
		4	6	1.87
		5	0	0.00
		6	10	3.13
		7	0	0.00
	□ = 18 ÷ 3	1	272	86.90
		2	22	7.03
		3	5	1.60
		4	1	0.32
		5	0	0.00
		6	12	3.83
		7	0	0.00

		Response	Number Giving This Response	Percent
Cell 12 (Cont.)	□ = 54 ÷ 6	1	260	83.07
		2	25	7.99
		3	3	0.96
		4	7	2.24
		5	0	0.00
		6	16	5.11
		7	0	0.00
	□ = 16 ÷ 8	1	271	86.58
		2	13	4.15
		3	2	0.64
		4	2	0.64
		5	0	0.00
		6	23	7.35
		7	0	0.00
	□ = 63 ÷ 7	1	255	77.74
		2	44	13.41
		3	1	0.30
		4	12	3.66
		5	0	0.00
		6	11	3.35
		7	0	0.00
	□ = 18 ÷ 9	1	264	80.49
		2	22	6.71
		3	1	0.30
		4	1	0.30
		5	0	0.00
		6	34	10.37
		7	0	0.00
	□ = 20 ÷ 4	1	299	88.72
		2	17	5.04
		3	4	1.19
		4	6	1.78
		5	0	0.00
		6	10	2.97
		7	0	0.00
	□ = 56 ÷ 7	1	274	81.31
		2	25	7.42
		3	0	0.00
		4	22	6.53
		5	0	0.00
		6	10	2.97
		7	0	0.00

Appendix G

PERCENT OF CODE 3, 4, 6, AND 7 RESPONSES ON NUMBER PUZZLE TEST (WHOLE NUMBER SOLUTION)

Cell	Example									
	1	2	3	4	5	6	7	8	9	10
3. a x b = □	0	0	0	0	0	1	0	0	6	0
6. □ = a x b	0	5	0	0	0	3	0	0	0	0
9. a ÷ b = □	5	0	1	0	0	2	6	0	-	-
12. □ = a ÷ b	0	2	1	1	0	0	1	0	-	-

Code 3 indicated the subject used the inverse operation, i.e., multiplied while solving a division open sentence or vice versa.

Cell	Example									
	1	2	3	4	5	6	7	8	9	10
1. □ x b = c	3	1	2	1	0	2	2	-	-	-
2 a x □ = c	0	1	0	0	4	1	0	-	-	-
3. a x b = □	0	3	1	7	3	2	8	3	0	2
4. c = □ x b	1	3	2	0	1	2	6	-	-	-
5. c = a x □	1	1	10	5	0	2	2	-	-	-
6. □ = a x b	2	0	0	0	4	1	2	1	0	2
7. □ ÷ b = c	0	0	0	0	1	0	1	1	1	-
8. a ÷ □ = c	0	2	3	0	6	4	0	-	-	-
9. a ÷ b = □	0	1	3	1	2	3	0	1	-	-
10. c = □ ÷ b	1	0	0	0	1	0	0	0	0	-
11. c = a ÷ □	0	1	4	1	3	1	2	-	-	-
12. □ = a ÷ b	2	0	2	1	4	0	2	7	-	-

Code 4 indicated the subject responded with a numeral quite close to the correct solution.

Cell	Example									
	1	2	3	4	5	6	7	8	9	10
1. □ x b = c	3	2	6	2	2	5	3	-	-	-
2. a x □ = c	3	3	4	4	2	2	3	-	-	-
3. a x b = □	1	3	2	5	4	4	3	1	1	4
4. c = □ x b	7	4	3	5	5	4	5	-	-	-
5. c = a x □	6	4	2	4	2	4	3	-	-	-
6. □ = a x b	2	2	2	1	1	1	2	2	2	3
7. □ ÷ b = c	11	3	8	8	6	5	4	10	9	-
8. a ÷ □ = c	4	6	3	4	4	1	2	-	-	-
9. a ÷ b = □	9	8	5	4	5	10	5	4	-	-
10. c = □ ÷ b	11	12	9	9	9	6	3	8	10	-
11. c = a ÷ □	7	4	2	3	5	6	3	-	-	-
12. □ = a ÷ b	3	4	5	7	3	10	3	3	-	-

Code 6 indicated the subject responded with some numeral not falling into any of the other six categories.

Cell	Example									
	1	2	3	4	5	6	7	8	9	10
8. a ÷ □ = c	0	0	0	1	0	1	1	-	-	-
11. c = a ÷ □	9	2	0	0	0	0	0	-	-	-

Code 7 indicated the subject performed the inverse operation across the equality sign. For example the open sentence 15 ÷ □ = 3 received 45 for the solution, indicating the subject might have changed the open sentence to the type 3 x 15 = □.

Appendix H

NO WHOLE-NUMBER SOLUTION CELLS

		Response	Number Giving This Response	Percent
Cell 1 MLA	□ x 5 = 16	1	300	89.02
		2	0	0.00
		3	0	0.00
		4	21	6.23
		5	0	0.00
		6	9	2.67
		7	0	0.00
	□ x 3 = 23	1	256	81.79
		2	0	0.00
		3	0	0.00
		4	20	6.39
		5	1	0.32
		6	34	10.86
		7	0	0.00
Cell 2 MLB	4 x □ = 27	1	249	77.81
		2	0	0.00
		3	0	0.00
		4	24	7.50
		5	0	0.00
		6	43	13.44
		7	0	0.00
	6 x □ = 43	1	269	82.01
		2	0	0.00
		3	0	0.00
		4	33	10.06
		5	0	0.00
		6	18	5.49
		7	0	0.00
Cell 4 MRA	21 = □ x 4	1	267	85.30
		2	0	0.00
		3	0	0.00
		4	19	6.07
		5	0	0.00
		6	25	7.99
		7	0	0.00
	29 = □ x 4	1	284	84.27
		2	0	0.00
		3	0	0.00
		4	24	7.12
		5	1	0.30
		6	21	6.23
		7	0	0.00

		Response	Number Giving This Response	Percent
Cell 5 MRB	20 = 7 x □	1	283	86.28
		2	0	0.00
		3	0	0.00
		4	16	4.88
		5	0	0.00
		6	21	6.40
		7	0	0.00
	47 = 8 x □	1	262	81.87
		2	0	0.00
		3	0	0.00
		4	26	8.12
		5	0	0.00
		6	28	8.75
		7	0	0.00
Cell 8 DLB	35 ÷ □ = 9	1	284	84.27
		2	0	0.00
		3	0	0.00
		4	8	2.37
		5	0	0.00
		6	38	11.28
		7	1	0.30
	47 ÷ □ = 6	1	258	82.43
		2	0	0.00
		3	0	0.00
		4	17	5.43
		5	0	0.00
		6	37	11.82
		7	1	0.32
Cell 9 DLC	13 ÷ 4 = □	1	274	85.62
		2	0	0.00
		3	0	0.00
		4	22	6.87
		5	0	0.00
		6	16	5.00
		7	0	0.00
	41 ÷ 8 = □	1	265	80.79
		2	0	0.00
		3	0	0.00
		4	23	7.01
		5	0	0.00
		6	29	8.84
		7	0	0.00

		Response	Number Giving This Response	Percent
Cell 11 DRB	3 = 28 ÷ □	1	245	78.27
		2	0	0.00
		3	0	0.00
		4	32	10.22
		5	0	0.00
		6	29	9.27
		7	3	0.96
	3 = 7 ÷ □	1	203	60.24
		2	0	0.00
		3	0	0.00
		4	8	2.37
		5	0	0.00
		6	20	5.93
		7	101	29.97
Cell 12 DRC	□ = 20 ÷ 3	1	259	80.94
		2	0	0.00
		3	9	2.81
		4	9	2.81
		5	0	0.00
		6	39	12.19
		7	0	0.00
	□ = 25 ÷ 4	1	269	82.01
		2	0	0.00
		3	4	1.22
		4	20	6.10
		5	0	0.00
		6	23	7.01
		7	0	0.00

Appendix I

PERCENT OF CODES 1 - 7 RESPONSES FOR THE NO WHOLE NUMBER SOLUTION OPEN SENTENCES

Cell	Percent of Code 1 Responses		Percent of Code 2 Responses		Percent of Code 3 Responses		Percent of Code 4 Responses	
	Examples 1	2	Examples 1	2	Examples 1	2	Examples 1	2
1. □ x b = c	89	82	0	0	0	0	6	6
2. a x □ = c	78	82	0	0	0	0	8	10
4. c = □ x b	85	84	0	0	0	0	6	7
5. c = a x □	86	82	0	0	0	0	5	8
8. a ÷ □ = c	84	82	0	0	0	0	2	5
9. a÷ b = □	86	81	0	0	0	0	7	7
11. c = a ÷ □	78	60	0	0	0	0	10	2
12. □ = a ÷ b	81	82	0	0	3	1	3	6

Cell	Percent of Code 5 Responses		Percent of Code 6 Responses		Percent of Code 7 Responses	
	Examples 1	2	Examples 1	2	Examples 1	2
1. $\square \times b = c$	0	0	3	11	0	0
2. $a \times \square = c$	0	0	13	5	0	0
4. $c = \square \times b$	0	0	8	6	0	0
5. $c = a \times \square$	0	0	6	9	0	0
8. $a \div \square = c$	0	0	12	12	0	0
9. $a \div b = \square$	0	0	5	9	0	0
11. $c = a \div \square$	0	0	9	6	1	30
12. $\square = a \div b$	0	0	12	7	0	0

Appendix J

CELL ENTRIES - RAW DATA MATRIX

Class 141 - 22 Students					
		A	B	C	D
Cell	1	12	2	14	7
	2	6	11	5	14
	3	18	7	9	21
	4	12	4	15	7
	5	5	12	5	14
	6	17	7	10	21
	7	17	7	10	21
	8	8	7	8	8
	9	10	8	8	7
	10	12	7	4	20
	11	4	2	2	5
	12	11	7	4	20
No. of Ob.		6	4	5	7

Class 142 - 21 Students					
		A	B	C	D
Cell	1	10	6	12	4
	2	5	17	4	12
	3	15	12	8	18
	4	10	5	12	5
	5	4	16	4	12
	6	15	10	8	18
	7	6	3	7	3
	8	8	12	7	6
	9	9	12	4	15
	10	2	1	5	2
	11	8	12	7	5
	12	6	12	4	15
No. of Ob.		5	6	4	6

Class 143 - 23 Students					
		A	B	C	D
Cell	1	8	7	20	4
	2	5	20	7	8
	3	15	13	12	11
	4	9	7	21	4
	5	4	20	7	7
	6	15	12	14	12
	7	8	2	8	6
	8	9	14	14	4
	9	9	14	7	9
	10	3	2	4	3
	11	9	14	12	4
	12	7	14	7	12
No. of Ob.		5	7	7	4

Class 144 - 19 Students					
		A	B	C	D
Cell	1	8	5	14	4
	2	5	15	4	7
	3	14	8	9	11
	4	9	4	15	4
	5	5	13	5	7
	6	15	10	8	12
	7	2	2	4	2
	8	9	9	9	4
	9	8	10	5	11
	10	1	2	4	3
	11	9	9	9	4
	12	10	9	4	11
No. of Ob.		5	5	5	4

Class 151 - 26 Students					
		A	B	C	D
Cell	1	13	6	21	5
	2	7	18	7	10
	3	20	12	12	12
	4	14	6	21	5
	5	6	19	7	10
	6	18	13	14	15
	7	2	7	10	2
	8	13	14	14	4
	9	14	12	7	14
	10	1	3	5	3
	11	12	12	14	5
	12	11	10	6	13
No. of Ob.		7	7	7	5

Class 152 - 27 Students					
		A	B	C	D
Cell	1	14	6	17	8
	2	7	18	6	16
	3	21	11	12	23
	4	14	6	16	8
	5	6	14	5	15
	6	21	12	9	24
	7	8	8	12	10
	8	14	12	10	8
	9	12	11	6	21
	10	4	2	3	4
	11	13	12	10	8
	12	10	12	6	23
No. of Ob.		7	6	6	8

Class 153 - 23 Students					
		A	B	C	D
Cell	1	16	6	17	3
	2	8	17	6	6
	3	24	11	12	7
	4	16	6	17	3
	5	8	18	6	6
	6	22	12	11	9
	7	7	7	9	2
	8	16	12	11	3
	9	14	12	6	9
	10	7	4	7	0
	11	11	12	11	3
	12	15	12	6	8
No. of Ob.		8	6	6	3

Class 162 - 29 Students					
		A	B	C	D
Cell	1	12	10	24	3
	2	6	33	8	6
	3	18	19	16	8
	4	11	11	24	3
	5	6	30	8	6
	6	18	21	16	9
	7	7	8	11	2
	8	12	21	15	3
	9	10	21	8	8
	10	7	7	7	1
	11	11	22	15	2
	12	12	18	8	9
No. of Ob.		7	11	8	3

Class 161 - 26 Students					
		A	B	C	D
Cell	1	17	3	16	7
	2	9	12	6	13
	3	27	7	12	20
	4	18	4	17	6
	5	8	11	6	14
	6	25	8	12	21
	7	8	6	6	8
	8	16	8	12	7
	9	17	7	4	18
	10	6	6	3	6
	11	17	7	10	7
	12	17	7	5	19
No. of Ob.		9	4	6	7

Class 163 - 32 Students					
		A	B	C	D
Cell	1	11	7	22	11
	2	6	21	7	20
	3	18	14	15	33
	4	12	7	20	10
	5	6	20	8	20
	6	18	14	15	32
	7	8	8	16	12
	8	12	14	15	11
	9	11	14	8	31
	10	5	6	11	9
	11	11	14	14	11
	12	10	14	8	31
No. of Ob.		6	7	8	11

Class 241 - 19 Students					
		A	B	C	D
Cell	1	8	5	18	2
	2	4	17	6	4
	3	11	11	12	6
	4	8	6	17	2
	5	3	15	6	4
	6	9	12	10	6
	7	2	6	4	1
	8	6	11	12	2
	9	5	11	6	6
	10	0	2	3	1
	11	8	12	11	2
	12	6	11	6	5
No. of Ob.		5	6	6	2

Class 251 - 22 Students					
		A	B	C	D
Cell	1	11	6	16	4
	2	5	18	6	8
	3	18	11	12	12
	4	10	6	18	4
	5	6	17	6	8
	6	18	11	10	12
	7	7	7	12	6
	8	11	12	12	4
	9	10	12	6	9
	10	6	6	9	6
	11	10	12	12	4
	12	11	11	6	11
No. of Ob.		6	6	6	4

Class 261 - 27 Students					
		A	B	C	D
Cell	1	12	8	15	7
	2	6	23	6	13
	3	17	14	12	20
	4	12	7	18	6
	5	6	20	6	13
	6	17	16	12	21
	7	8	9	7	3
	8	11	16	11	7
	9	10	16	6	21
	10	3	6	4	2
	11	11	15	11	7
	12	11	13	6	20
No. of Ob.		6	8	6	7

Class 242 - 20 Students					
		A	B	C	D
Cell	1	8	4	15	5
	2	4	14	5	11
	3	12	9	10	18
	4	8	5	15	3
	5	3	14	5	7
	6	12	10	10	16
	7	3	1	7	6
	8	8	10	8	5
	9	8	10	5	15
	10	0	0	5	5
	11	7	8	7	5
	12	8	9	4	16
No. of Ob.		4	5	5	6

Class 252 - 25 Students					
		A	B	C	D
Cell	1	12	5	20	6
	2	6	14	7	14
	3	18	10	14	19
	4	12	5	19	5
	5	6	15	7	12
	6	17	10	14	21
	7	7	8	13	5
	8	11	9	14	6
	9	9	10	7	21
	10	6	6	7	2
	11	11	10	11	6
	12	9	9	7	19
No. of Ob.		6	5	7	7

Class 262 - 27 Students					
		A	B	C	D
Cell	1	13	7	21	6
	2	7	21	6	12
	3	21	14	13	17
	4	14	7	18	5
	5	6	19	7	12
	6	21	13	14	18
	7	8	9	11	8
	8	12	12	13	6
	9	8	14	7	18
	10	6	8	7	8
	11	13	12	14	6
	12	12	14	6	18
No. of Ob.		7	7	7	6

Class 341 - 19 Students					
		A	B	C	D
Cell	1	10	4	13	4
	2	5	12	5	9
	3	12	6	9	15
	4	10	2	14	4
	5	4	11	5	7
	6	11	8	9	15
	7	6	4	0	6
	8	10	7	9	5
	9	9	8	5	13
	10	2	3	1	3
	11	8	8	7	3
	12	6	6	5	7
No. of Ob.		5	4	5	5

Class 342 - 11 Students					
		A	B	C	D
Cell	1	6	4	6	2
	2	3	12	2	3
	3	9	7	4	6
	4	5	4	6	1
	5	3	10	2	4
	6	9	8	4	6
	7	3	2	1	1
	8	6	8	4	2
	9	5	8	2	5
	10	2	2	1	0
	11	4	8	4	1
	12	6	8	2	5
No. of Ob.		3	4	2	2

Class 351 - 13 Students					
		A	B	C	D
Cell	1	6	3	15	2
	2	3	9	5	4
	3	9	6	9	6
	4	6	3	15	2
	5	3	8	5	4
	6	9	6	10	6
	7	3	2	11	2
	8	6	6	10	2
	9	4	6	4	5
	10	3	0	7	2
	11	6	6	9	2
	12	4	6	5	6
No. of Ob.		3	3	5	2

Class 352 - 28 Students					
		A	B	C	D
Cell	1	18	4	17	9
	2	9	12	6	18
	3	26	7	11	26
	4	17	4	15	9
	5	9	12	5	18
	6	27	8	12	27
	7	10	5	7	16
	8	17	8	11	9
	9	17	8	6	26
	10	11	5	8	12
	11	18	8	10	8
	12	18	8	6	23
No. of Ob.		9	4	6	9

Class 361 - 28 Students					
		A	B	C	D
Cell	1	16	4	21	6
	2	8	18	6	14
	3	24	12	13	19
	4	15	6	19	5
	5	8	17	7	13
	6	24	12	14	20
	7	11	6	14	3
	8	13	12	14	7
	9	12	10	7	18
	10	8	6	12	2
	11	15	12	13	7
	12	14	12	7	20
No. of Ob.		8	6	7	7

Class 362 - 25 Students					
		A	B	C	D
Cell	1	10	7	23	5
	2	4	21	7	10
	3	15	14	14	13
	4	10	7	21	5
	5	5	20	7	10
	6	14	14	16	15
	7	3	4	19	7
	8	9	13	14	5
	9	9	14	6	14
	10	4	0	14	6
	11	9	13	14	5
	12	8	14	5	15
No. of Ob.		5	7	8	5

Class 363 - 29 Students					
		A	B	C	D
Cell	1	10	10	24	5
	2	5	29	8	12
	3	15	19	16	18
	4	10	9	24	6
	5	5	29	8	12
	6	15	20	16	18
	7	5	14	20	8
	8	9	19	16	5
	9	10	20	8	17
	10	3	18	17	6
	11	9	19	16	6
	12	10	18	8	18
No. of Ob.		5	10	8	6

Class 441 - 12 Students

		A	B	C	D
Cell	1	8	4	5	2
	2	4	11	2	4
	3	12	8	3	6
	4	6	3	3	2
	5	3	11	2	4
	6	12	8	2	6
	7	7	1	4	1
	8	7	6	4	2
	9	6	8	2	5
	10	2	0	1	1
	11	5	8	3	2
	12	6	6	2	5
No. of Ob.		4	4	2	2

Class 461 - 16 Students

		A	B	C	D
Cell	1	10	3	12	4
	2	5	8	4	8
	3	15	6	8	12
	4	10	3	12	4
	5	5	9	4	6
	6	15	6	8	12
	7	4	2	6	4
	8	10	5	3	4
	9	10	6	4	12
	10	3	2	7	2
	11	9	6	6	4
	12	9	6	4	12
No. of Ob.		5	3	4	4

Class 542 - 26 Students

		A	B	C	D
Cell	1	12	7	16	7
	2	6	20	6	14
	3	17	13	12	19
	4	12	6	17	5
	5	5	19	6	14
	6	18	14	10	20
	7	6	6	10	9
	8	12	13	10	7
	9	12	14	6	17
	10	2	3	5	2
	11	7	14	11	6
	12	5	13	6	17
No. of Ob.		6	7	6	7

Class 451 - 10 Students

		A	B	C	D
Cell	1	4	3	3	3
	2	2	9	2	6
	3	6	6	3	9
	4	4	3	4	3
	5	2	9	1	6
	6	5	6	1	9
	7	3	4	2	5
	8	4	6	3	3
	9	2	6	1	9
	10	0	5	1	5
	11	4	4	2	3
	12	4	5	1	8
No. of Ob.		2	3	2	3

Class 541 - 27 Students

		A	B	C	D
Cell	1	12	5	19	5
	2	6	22	8	9
	3	18	16	14	15
	4	12	8	22	5
	5	5	21	8	10
	6	16	16	16	15
	7	4	11	7	5
	8	12	16	12	5
	9	11	15	7	13
	10	4	9	6	7
	11	7	16	14	5
	12	11	15	7	15
No. of Ob.		6	8	8	5

Class 551 - 15 Students

		A	B	C	D
Cell	1	9	4	12	1
	2	5	15	3	2
	3	14	10	7	3
	4	10	5	11	1
	5	5	14	4	2
	6	15	10	7	3
	7	2	8	9	2
	8	10	10	5	1
	9	7	10	3	2
	10	3	4	5	0
	11	8	10	8	1
	12	6	9	4	3
No. of Ob.		5	5	4	1

Class 552 - 22 Students

		A	B	C	D
Cell	1	10	7	17	4
	2	5	21	6	8
	3	15	14	11	12
	4	9	6	16	3
	5	5	21	6	8
	6	14	14	11	12
	7	3	10	9	6
	8	10	14	12	4
	9	10	13	6	12
	10	3	8	7	5
	11	9	13	12	4
	12	10	14	6	9
No. of Ob.		5	7	6	4

Class 561 - 26 Students

		A	B	C	D
Cell	1	14	5	21	6
	2	7	17	7	12
	3	21	11	12	17
	4	13	5	20	5
	5	7	15	7	11
	6	21	10	14	17
	7	8	9	13	5
	8	14	11	14	6
	9	14	12	7	15
	10	6	9	10	6
	11	13	11	14	6
	12	13	10	7	17
No. of Ob.		7	6	7	6

Class 642 - 21 Students

		A	B	C	D
Cell	1	11	6	10	4
	2	6	18	4	9
	3	17	9	7	14
	4	11	6	10	4
	5	6	18	4	8
	6	18	11	6	14
	7	2	5	6	0
	8	12	12	6	5
	9	10	10	3	14
	10	3	3	3	0
	11	8	11	6	5
	12	12	11	4	12
No. of Ob.		6	6	4	5

Class 561 - 24 Students

		A	B	C	D
Cell	1	10	7	14	7
	2	5	21	5	14
	3	15	13	9	20
	4	10	7	15	7
	5	5	19	5	13
	6	15	14	10	21
	7	6	6	7	8
	8	9	12	10	7
	9	10	14	5	20
	10	3	2	8	4
	11	9	14	10	6
	12	10	12	5	21
No. of Ob.		5	7	5	7

Class 641 - 17 Students

		A	B	C	D
Cell	1	6	3	9	4
	2	4	12	4	10
	3	12	8	7	15
	4	8	4	12	3
	5	4	11	4	9
	6	12	6	8	15
	7	3	4	4	4
	8	6	4	6	5
	9	7	7	3	14
	10	2	0	1	0
	11	6	2	6	4
	12	6	6	4	13
No. of Ob.		4	4	4	5

Class 643 - 20 Students

		A	B	C	D
Cell	1	8	5	16	5
	2	4	15	6	10
	3	12	10	11	15
	4	6	5	18	5
	5	3	12	6	10
	6	9	9	12	15
	7	2	5	4	8
	8	8	10	11	5
	9	8	9	5	15
	10	2	6	5	6
	11	3	9	12	5
	12	6	9	5	15
No. of Ob.		4	5	6	5

Class 651 - 26 Students					
		A	B	C	D
Cell	1	16	6	14	6
	2	8	20	5	12
	3	24	14	9	18
	4	14	5	14	6
	5	8	18	5	11
	6	24	14	10	18
	7	9	3	4	7
	8	15	13	9	6
	9	14	12	5	16
	10	4	1	4	3
	11	12	12	10	6
	12	14	13	5	16
No. of Ob.		8	7	5	6

Class 652 - 22 Students					
		A	B	C	D
Cell	1	12	5	15	4
	2	6	21	5	7
	3	17	14	9	12
	4	10	7	15	4
	5	6	20	5	8
	6	18	14	10	12
	7	6	8	6	4
	8	12	14	8	4
	9	12	14	5	8
	10	3	5	5	1
	11	9	14	10	3
	12	9	13	4	12
No. of Ob.		6	7	5	4

Class 653 - 23 Students					
		A	B	C	D
Cell	1	10	5	16	7
	2	4	15	6	14
	3	15	9	11	19
	4	10	5	16	6
	5	5	13	6	14
	6	15	9	12	21
	7	1	5	7	7
	8	10	9	9	7
	9	9	9	6	20
	10	0	2	7	1
	11	6	9	11	7
	12	7	9	6	20
No. of Ob.		5	5	6	7

Class 661 - 23 Students					
		A	B	C	D
Cell	1	11	7	12	6
	2	6	21	3	12
	3	17	12	7	18
	4	11	7	11	6
	5	5	20	4	11
	6	17	14	8	18
	7	9	11	3	8
	8	11	14	8	6
	9	11	14	3	18
	10	7	6	6	6
	11	12	13	7	6
	12	10	13	4	18
No. of Ob.		6	7	4	6

Class 662 - 28 Students					
		A	B	C	D
Cell	1	15	6	21	7
	2	7	17	7	13
	3	24	10	13	20
	4	16	6	19	6
	5	8	17	7	14
	6	23	12	14	21
	7	7	10	11	9
	8	13	12	13	7
	9	14	12	5	21
	10	8	10	6	9
	11	16	12	13	7
	12	15	12	7	19
No. of Ob.		8	6	7	7

Class 663 - 22 Students					
		A	B	C	D
Cell	1	12	3	19	5
	2	6	10	6	10
	3	18	6	12	15
	4	10	4	20	5
	5	6	9	7	10
	6	16	8	14	15
	7	4	7	10	7
	8	10	8	13	5
	9	11	7	7	14
	10	0	3	9	2
	11	9	8	13	5
	12	10	7	7	12
No. of Ob.		6	4	7	5

Class 741 - 9 Students					
		A	B	C	D
Cell	1	3	1	3	2
	2	2	2	1	2
	3	6	2	2	3
	4	1	1	3	1
	5	2	2	1	2
	6	5	2	2	3
	7	2	2	3	0
	8	2	1	2	1
	9	2	2	1	2
	10	0	0	0	0
	11	2	2	2	1
	12	1	2	1	3
No. of Ob.		2	1	1	2

Class 761 - 21 Students					
		A	B	C	D
Cell	1	10	3	13	7
	2	5	11	5	14
	3	14	6	10	21
	4	9	3	13	3
	5	4	9	5	9
	6	14	8	10	18
	7	5	2	4	2
	8	8	8	6	7
	9	8	7	5	19
	10	5	1	4	2
	11	9	6	6	6
	12	9	6	4	15
No. of Ob.		5	4	5	7

Class 842 - 24 Students					
		A	B	C	D
Cell	1	10	5	18	8
	2	5	14	6	14
	3	15	10	11	20
	4	8	5	16	8
	5	5	14	6	14
	6	13	10	12	24
	7	4	4	6	11
	8	7	10	12	8
	9	9	10	6	22
	10	1	0	3	7
	11	9	9	12	8
	12	8	8	6	23
No. of Ob.		5	5	6	8

Class 751 - 9 Students					
		A	B	C	D
Cell	1	4	2	5	2
	2	2	6	1	4
	3	6	4	3	6
	4	3	2	4	1
	5	1	6	2	4
	6	6	4	4	6
	7	1	4	3	4
	8	4	4	3	2
	9	2	4	0	5
	10	0	4	2	1
	11	3	4	2	1
	12	2	4	2	4
No. of Ob.		2	3	2	2

Class 841 - 13 Students					
		A	B	C	D
Cell	1	6	4	9	2
	2	3	12	3	6
	3	9	8	5	9
	4	6	4	8	2
	5	3	12	3	5
	6	9	8	6	9
	7	1	3	6	1
	8	6	8	5	3
	9	6	8	3	9
	10	0	2	3	0
	11	5	6	5	2
	12	6	8	3	7
No. of Ob.		3	4	3	3

Class 843 - 25 Students					
		A	B	C	D
Cell	1	14	6	17	4
	2	8	17	6	8
	3	24	12	12	14
	4	14	6	17	5
	5	7	16	6	8
	6	24	12	12	15
	7	8	1	3	6
	8	14	12	12	5
	9	12	11	6	12
	10	6	0	3	3
	11	10	12	11	4
	12	12	11	5	15
No. of Ob.		8	6	6	5

Class 863 - 21 Students

		A	B	C	D
Cell	1	9	4	20	5
	2	5	12	7	10
	3	15	8	13	13
	4	10	4	21	4
	5	4	11	7	10
	6	14	8	13	15
	7	5	6	10	1
	8	10	8	12	5
	9	7	7	7	14
	10	5	5	7	2
	11	10	8	11	5
	12	10	7	7	13
No. of Ob.		5	4	7	5

Class 852 - 21 Students

		A	B	C	D
Cell	1	12	4	18	4
	2	6	15	6	8
	3	17	10	12	11
	4	12	5	18	4
	5	4	15	6	8
	6	16	10	11	10
	7	3	6	7	2
	8	11	9	11	4
	9	12	10	4	12
	10	4	3	5	1
	11	9	8	10	3
	12	10	8	6	12
No. of Ob.		6	5	6	4

Class 861 - 8 Students

		A	B	C	D
Cell	1	8	2	3	1
	2	4	6	1	2
	3	12	4	2	3
	4	8	2	3	1
	5	4	6	1	2
	6	11	4	2	3
	7	6	4	0	1
	8	8	3	2	1
	9	7	4	1	2
	10	6	4	1	1
	11	8	4	2	1
	12	8	3	1	3
No. of Ob.		4	2	1	1

Class 851 - 16 Students

		A	B	C	D
Cell	1	5	5	11	4
	2	3	15	4	8
	3	9	9	7	12
	4	6	5	12	3
	5	3	14	4	8
	6	9	10	8	12
	7	2	7	12	5
	8	6	9	7	4
	9	6	10	4	12
	10	0	6	12	4
	11	6	10	8	4
	12	6	9	4	12
No. of Ob.		3	5	4	4

Class 853 - 18 Students

		A	B	C	D
Cell	1	10	2	15	5
	2	5	9	5	10
	3	15	5	9	13
	4	9	2	15	5
	5	4	7	5	9
	6	13	6	10	15
	7	5	2	6	9
	8	9	5	10	4
	9	10	4	5	14
	10	2	0	4	4
	11	5	5	10	4
	12	7	3	5	13
No. of Ob.		5	3	5	5

Class 862 - 23 Students

		A	B	C	D
Cell	1	8	5	18	7
	2	4	17	6	13
	3	12	11	11	21
	4	6	5	18	6
	5	4	17	6	14
	6	12	12	11	19
	7	5	7	8	7
	8	8	12	12	7
	9	7	12	6	20
	10	3	5	10	5
	11	6	12	12	6
	12	7	12	5	17
No. of Ob.		4	6	6	7

Appendix K

TRANSFORMATION MATRIX

Cell	Test Form			
	A	B	C	D
Cell 1	3	6	2	6
Cell 2	6	2	6	3
Cell 3	2	3	3	2
Cell 4	3	6	2	6
Cell 5	6	2	6	3
Cell 6	2	3	3	2
Cell 7	3	3	2	3
Cell 8	3	3	3	6
Cell 9	3	3	6	2
Cell 10	3	3	2	3
Cell 11	3	3	3	6
Cell 12	3	3	6	2

Number of Sentence Types Per Test Form Contributing to Each Cell

Cell	Test Form			
	A	B	C	D
Cell 1	2	1	3	1
Cell 2	1	3	1	2
Cell 3	3	2	2	3
Cell 4	2	1	3	1
Cell 5	1	3	1	2
Cell 6	3	2	2	3
Cell 7	2	2	3	2
Cell 8	2	2	2	1
Cell 9	2	2	1	3
Cell 10	2	2	3	2
Cell 11	2	2	2	1
Cell 12	2	2	1	3

Appendix L

ANOVA--RESULTS USING a VS b AND ab VS c
AS COMPARED WITH b VS c AND bc VS a

	DF	MS	F	P
Placeholder	2	164.1808	903.3201	.0001
a vs b	1	249.8057	1325.8050	.0001
ab vs c	1	78.5559	480.8352	.0001
b vs c	1	.0515	.7873	.3792
bc vs a	1	328.3101	1146.2927	.0001
Grade and Place	4	1.3578	7.7182	.0012
a vs b	2	1.4582	7.7392	.0012
ab vs c	2	1.2575	7.6972	.0013
b vs c	2	.1745	2.6694	.0792
bc vs a	2	2.5412	8.8726	.0006
MD x Place	2	138.0383	667.4437	.0001
MD x a vs b	1	211.6520	944.6439	.0001
MD x ab vs c	1	64.4246	390.2435	.0001
MD x b vs c	1	.1043	.5030	.2260
MD x bc vs a	1	275.9722	863.1252	.0001
MD x Place x G	4	.7256	3.7465	NS
MD x a vs b	2	.8140	3.6332	.0337
MD x ab vs c	2	.6372	3.8598	.0277
MD x b vs c	2	.0580	.8360	.4395
MD x bc vs a	2	1.3932	4.3574	.0181
LR x P	2	4.2760	57.1078	.0001
LR x a vs b	1	3.0432	20.0365	.0001
LR x ab vs c	1	5.5089	94.1791	.0001
LR x b vs c	1	1.3466	29.1088	.0001
LR x bc vs a	1	7.2055	43.9048	.0001
LR x P x Grade	4	.0593	.9044	NS
LR x a vs b	2	.0208	.1371	.8723
LR x ab vs c	2	.0978	1.6716	.1983
LR x b vs c	2	.0671	1.4515	.2440
LR x bc vs a	2	.0515	.3135	.7323
MD x LR x P	2	2.2694	26.5586	.0001
Above with ab	1	3.9553	44.3866	.0001
Above x ab vs c	1	.5835	8.7305	.0048
Above x bc	1	.1108	2.1485	.1490
Above x bc vs a	1	4.4280	42.4215	.0001
MD x LR x P x G	4	.1158	1.6164	NS
Above x ab	2	.0619	.6943	.5042
Above x ab vs c	2	.1697	2.5376	.0892
Above x bc	2	.0546	1.0588	.3546
Above x bc vs a	2	.1769	1.6951	.1940

REFERENCES

Cromer, F. E. Structural models for predicting the difficulty of multiplication problems. Journal for Research in Mathematics Education, 1974, 5: 155-166.

Davis, R. B. Discovery in mathematics. Reading, Massachusetts: Addison-Wesley, 1964.

Finn, J. D. Multivariance: Fortran program for univariate and multivariate analysis of variance and covariance. Buffalo, New York: Department of Educational Psychology, State University of New York at Buffalo, 1967.

Grouws, D. A. Differential performance of third-grade children in solving open sentences of four types. (Doctoral dissertation, University of Wisconsin) Madison, Wisconsin: 1971.

Grouws, D. A. Open sentences: Some instructional considerations from research. Arithmetic Teacher, 1972, 19, 595-599.

Hays, W. L. Statistics for psychologists. New York: Holt, Rinehart and Winston, 1965.

Heddens, J. W. Todays mathematics. Chicago: Science Research Associates, 1968.

Kirk, R. E. Experimental design: Procedures for the behavioral sciences. Belmont, California: Brooks/Cole, 1968.

Osburn, Worth J. Levels of difficulty in long division. Elementary School Journal, 1950, 46, 441-447.

Suppes, P. & Morningstar, M. Computer-assisted instruction at Stanford, 1966-68: Data, models, and evaluation of the arithmetic programs. New York: Academic Press, 1972.

Suydam, M. N. & Weaver, J. F. Multiplication and division with whole numbers. Set-B, H 2.

Van Engen, H. Why use frames in arithmetic? Arithmetic Teacher, 1966, 13, 315-316.

Van Engen, H. & Gibb, E. G. General mental functions associated with division. Educational Service Studies, No. 2. Cedar Falls: Iowa. State Teachers College, 1956.

Weaver, J. F. Some factors associated with pupils' performance levels on simple open addition and subtraction sentences. Arithmetic Teacher, 1971, 18, 513-519.

Weaver, J. F. The symmetric property of the equality relation and young children's ability to solve open addition and subtraction sentences. Journal for Research in Mathematics Education, 1973, 4: 45-56.

Weaver, J. F. The ability of first-, second-, and third-grade pupils to identify open addition and subtraction sentences for which no solution exists within the set of whole numbers. School Science and Mathematics, 1972, 72, 679-691.

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